Computation and Visualization of Invariant Manifolds

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Abstract

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In this thesis, we start with the basic concepts of dynamical systems. Then we introduce the general types of problems that the well-known software package AUTO solves. AUTO uses a boundary value algorithm with Gaussian collocation and pseudo-arclength continuation. The two features distinguish AUTO from other general ODE solvers for dynamical systems. In order to compute 2D solution manifolds, AUTO uses orbit continuation. With these tools, we study two famous problems, the Lorenz system and the circular restricted Three-Body problem (CR3BP). We briefly discuss the basic bifurcation and stability analysis of ODE systems. The numerical analysis of the two problems leads to the newest algorithm to compute the 2D stable manifold of the origin of the Lorenz system and the 2D unstable manifold of appropriate periodic orbits of the CR3BP. We utilize Python for the flow control of AUTO. We also implement two visualization packages, QTPlaut and MATPlaut. They make possible the processing of large quantities of AUTO solution data with the OpenGL graphical library, dynamic memory allocation and interpolation methods. We conclude with prospect for future research.